# PVS 6.0 and Beyond

NASA/NIA PVS Class 2012

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## Contents

## Declaration Parameters

- PVS has theory level parameters, which allow generic theories to be defined
- They are very useful, and are extensively used in the prelude and NASA libraries
- But there are situations where they are not so convenient
- The NASA libraries introduce groups with

groups
```
groups_scaf[T: TYPE, *: [T,T -> T], one: T]: THEORY
```

- Homomorphisms require two sets of parameters, hence another theory:

homomorphisms
```
homomorphism_lemmas[T1: TYPE, *: [T1,T1 -> T1], one1: T1,
                    T2: TYPE, o: [T2,T2 -> T2], one2: T2]: THEORY
```

## Declaration Parameters (cont)

- A rather simple result in group theory is that homomorphisms are associative:

$$G_1 \xrightarrow{f} G_2 \xrightarrow{g} G_3 \xrightarrow{h} G_4 \supset (h \circ g) \circ f = h \circ (g \circ f)$$

- But this requires four sets of parameters, and is not included in the NASA library

## Declaration Parameters (cont)

To fix this, we introduce declaration level parameters
Illustrated with yet another group theory:

groups

```
groups[G: TYPE]: THEORY
BEGIN
 group: TYPE =
   [# e: G,
      P: {f: [G, G -> G] | associative?(f)
                               and forall (g: G): f(g, e) = g},
      M: {i: [G -> G] | forall (g: G): P(g, i(g)) = e} #]
END groups
```

## Declaration Parameters (cont)

group_morphisms

```
group_morphisms: THEORY
BEGIN
 importing groups
 homo?[G1, G2: TYPE](g1: group[G1], g2: group[G2])(f: [G1 -> G2]):
        bool =
   f(g1`e) = g2`e and
   forall (x, y: G1): f(g1`P(x, y)) = g2`P(f(x), f(y)) and
   forall (x: G1): f(g1`M(x)) = g2`M(f(x))

 homo[G1, G2: TYPE](g1: group[G1], g2: group[G2]): TYPE
       = (homo?[G1, G2](g1, g2))

 homo_is_assoc[G1, G2, G3, G4: TYPE]: lemma
   forall (g1: group[G1], g2: group[G2], g3: group[G3], g4: group[G4],
           f: (homo?[G1, G2](g1, g2)), g: (homo?[G2, G3](g2, g3)),
           h: (homo?[G3, G4](g3, g4))):
      h o (g o f) = (h o g) o f
END group_morphisms
```

## PVS as Why3 Backend

- Why3 is a software verification platform
- Features an ML-style language
- Interfaces to various automated and interactive theorem provers
- Some changes introduced in Why3 made it difficult to support PVS
- In principle, this could be done in PVS by refactoring, but it is difficult

## PVS as Why3 Backend (cont)

**Why3 to PVS**

```
...
% Why3 zwf_zero
zwf_zero(a:int, b:int): bool = (0 <= b) AND (a <  b)

% Why3 alloc_table
alloc_table[t:TYPE]: TYPE+
...
% Why3 memory
memory[t:TYPE, v:TYPE]: TYPE+

% Why3 select
select[t:TYPE, v:TYPE+](x:memory[t, v], x1:pointer[t]): v
...
% Why3 pset
pset[t:TYPE]: TYPE+

% Why3 pset_empty
pset_empty[t:TYPE]: pset[t]
...
```

## Declaration Parameters: Advanced Example

### Monads

```
monad: THEORY
 BEGIN

 m[a: TYPE]: TYPE

 return[a: TYPE]: [a -> m[a]]

 >>=[a, b: TYPE](x: m[a], f: [a -> m[b]]): m[b] % infix
 >>=[a, b: TYPE](x: m[a])(f: [a -> m[b]]): m[b] = x >>= f; % Curried

 >>[a, b: TYPE](x: m[a])(y: m[b]): m[b] = x >>= (lambda (z: a): y);

 join[a: TYPE](x: m[m[a]]): m[a] = x >>= id[m[a]]

 bind_return[a, b: TYPE]: AXIOM
   FORALL (x: a, f: [a -> m[b]]): (return[a](x) >>= f) = f(x)

 bind_ret2[a: TYPE]: AXIOM
   FORALL (x: m[a]): (x >>= return[a]) = x

END monad
```

## Monads continued

### Maybe Monad

```
Maybe[a: TYPE]: datatype
BEGIN
  Nothing: Nothing?
  Just(Val: a): Just?
END Maybe

maybe: THEORY
BEGIN
 IMPORTING Maybe
 IMPORTING monad
   {{ m[a: type] := Maybe[a],
      return[a: type] := Just[a],
      >>=[a, b: type](x:Maybe[a], f: [a -> Maybe[b]])
            := CASES x OF Nothing: Nothing,
                          Just(y): f(y) ENDCASES }}
 f(x: int): Maybe[int] =
      IF rem(2)(x) = 0 THEN Nothing ELSE Just(2 * x) ENDIF
 g(x: int): Maybe[int] =
      IF rem(3)(x) = 0 THEN Nothing else Just(3 * x) ENDIF
 h(x: int): Maybe[int] =
      IF rem(5)(x) = 0 THEN Nothing ELSE Just(5 * x) ENDIF
 k(x: int): Maybe[int] = f(x) >>= g >>= h
END maybe
```

## Expression Judgements

- PVS judgements work on types, names, numbers, and functions
- This has been extended, can now give types to arbitrary expressions under a universal quantifier:

Expression Judgements

```
expr_jdg: THEORY
BEGIN
 ej: JUDGEMENT FORALL (x: real): x*x HAS_TYPE nnreal
 f: [nnreal -> real]
 fm: FORMULA
   FORALL (y: real):
     f(f((y - 100) * (y - 100)) * f((y - 100) * (y - 100))) = 2
END expr_jdg
```

## Unicode

- Simple ASCII text is very limiting
- The Unicode standard extends this, providing a standard for representing over 110,000 characters
- Both Lisp and Emacs (among many other applications) have builtin support for Unicode
- In addition to display, Emacs has several *input methods* for conveniently inserting Unicode characters
- It was relatively simple to modify the PVS parser to allow Unicode characters

## Unicode Demo

- `M-x list-input-methods` list available input methods
- `M-x set-input-method` selects the (buffer specific) input method
- `M-x describe-input-method` shows how to input the characters
- `C-x 8 <RET>` inputs a character by name

## Unicode To Do

- Identify unary, binary, mix-fix, etc. operators to be included in the PVS grammar
- Unicode is difficult to directly use in `alltt`, hence needs translation
- Create a PVS input method to make it easy to insert frequently used symbols
- Backward compatibility could be supported

## Numeric Simplification

- Cesar requested better handling of numeric values in PVS
- We provided new internal representations that significantly sped up processing
- This was fairly limited, and a flag had to be set to enable it
- In PVS 6.0, the numeric operations (+, -, *, /) are simplified aggressively

## Numeric Simplification (cont)

```
dec :

  |-------
{1}   FORALL (u, s: real):
        u >= 0.78 AND s > 0 AND s < 4 AND u < 0.9 IMPLIES
         -(0.115210368 * s) - 0.101102976 * s - 0.15072 * s * u -
          0.1301216 * s * u
          - 0.018146688 * s
          - 0.4702464
          - 4 * (0.1296192 * u)
          + 0.404411904
          + 4 * (0.15072 * u)
          + 0.072586752
          + 0.1175616 * s
          + 0.101494848 * s
          + 0.015614592 * s
          + 0.1477056 * s * u
          + 0.1296192 * s * u
          >= 0
```

## Numeric Simplification (cont)

```
Rule? (assert)
Simplifying, rewriting, and recording with decision procedures,
this simplifies to:
dec :

  |-------
1   FORALL (u, s: real):
        u >= 0.78 AND s > 0 AND s < 4 AND u < 0.9 IMPLIES
         13188/1953125 - 1099/312500 * (s * u) + 3297/15625000 * s +
          6594/78125 * u
          >= 0
```

## What's ahead?

- New GUI Interface
- SMT-LIB integration
- Dimension checking
- Evidential Tool Bus (ETB)
- Kernel of Truth (KoT)

## New GUI Interface

- We are working on a new interface to PVS
- Roughly speaking, the current Emacs interface will be reimplemented as JSON
- The PVS Lisp image will act as a server
- Started trying to make an Eclipse interface
  - very difficult—PVS is **not** Java
- Now working on one based on wxPython

## PVS GUI

Demo

## SMT-LIB integration

- Yices and Yices2 are already integrated into PVS
- SMT-LIB (`http://www.smtlib.org/` provides
  - standard rigorous descriptions of background theories for Satisfiability Modulo Theory (SMT) solvers,
  - common input and output languages used for these theories,
  - a large library of benchmarks
- The PVS integration provides an `smt` prover rule
- This can invoke any SMT solver that can parse SMT-LIB, (Z3, CVC4, and others)
- The advantage is that any new features provided in an SMT solver are quickly available in PVS

## Dimension Checking

- DimSim is dimensional analysis extension to Simulink (with
- It checks that Simulink blocks are dimensionally consistent, using a form of Gauss-Jordan elimination
- Paper was presented at FM 2012
- The PVS language has been extended to include dimension types, and we plan on integrating the DimSim algorithm

## Formal Tool Integration

- Software and hardware designs are used in many critical applications
- Formal and semi-formal tools are used in analysis and synthesis at all phases of the design lifecycle.
- A typical project integrates many diverse tools
- How do we create systematic workflows that integrates multiple tools?
- How do we make these workflows replayable?

## Examples of Tool Integration

- Counterexample-guided abstraction refinement (CEGAR)
- Concolic execution uses symbolic evaluation with a SAT or SMT solver
- Bounded model checking employs a SAT or SMT solver
- Simplification using a computer algebra system such as REDUCE or QEPCAD
- Proof obligation generation for pre/post-condition specifications and refinement steps using the PVS type checker.
- Invariant generation using a range of techniques such as static analysis, templates, dynamic analysis, $k$-induction.
- Combining a verification condition generator with a range of deductive techniques for discharging proof obligations.
- Using a high-performance automated theorem prover to find an unsatisfiable core of input formulas

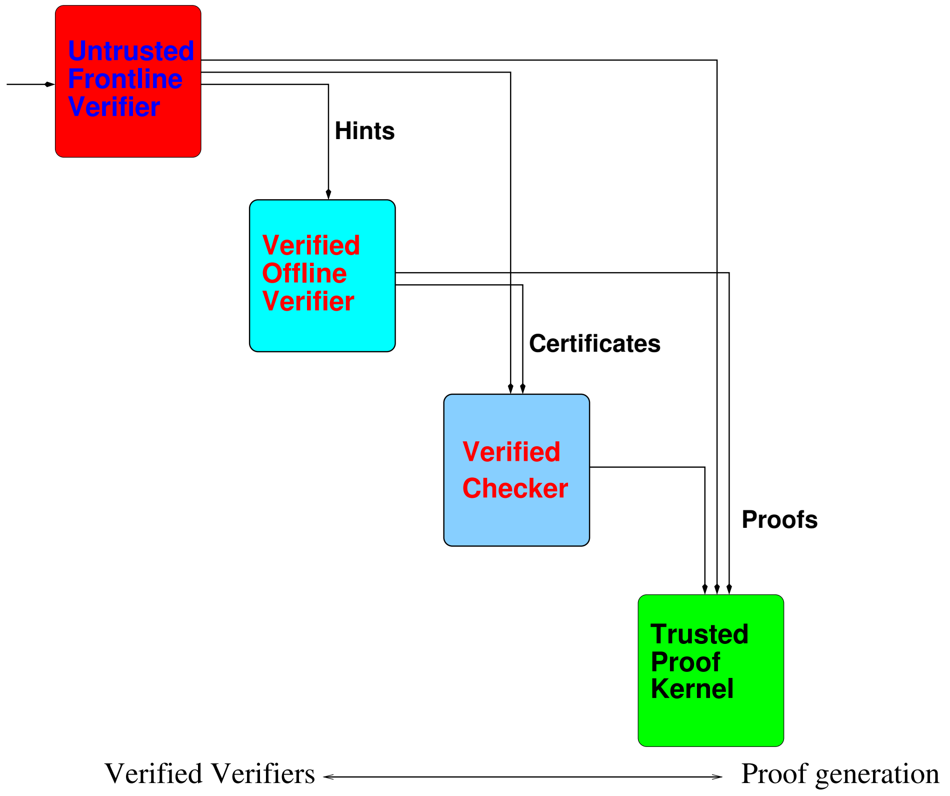
## Evidential Tool Bus (ETB)

- The main goal of ETB is the production of claims supported by arguments, where some sub-claims in the argument can be established by external tools.
- The ETB should extensible with new claim forms and rules of argumentation.
- The ETB should admit new external tools that interact with the ETB through an API to produce claims and generate queries.
- Workflows involving external tools should be definable as scripts.
- The argument produced by a completed development using the ETB should be checkable.
- The ETB explicates the tools and assumptions on which an argument depends.
- The ETB should be semantically neutral so that it does not exclude any tools, languages, or models.

## ETB Design Choices

- Information in the ETB is in the form of queries and claims which are either atoms or negations of atoms.
- An atom is an *n*-ary predicate applied to *n* arguments that are either data objects or variables.
- A claim is a ground (i.e., fully instantiated) atom or its negation that is asserted to hold.
- A query is a partially instantiated atom that triggers a chain of inference.
- Data objects are either JSON representations or tool or file handles.
- External tools are integrated into ETB through interpreted predicates.
- Scripts are Datalog programs defined through uninterpreted predicates.
- An ETB proof is a tree of claims where each claim follows from the subclaims by a rule of inference.
- An ETB instance is a network of server and client nodes

## Kernel of Truth



## The Kernel of Truth (KoT)

- The kernel contains a reference proof system formalizing ZFC.
- It also contains several verified checkers for specialized certificate formats.
- If the checker validates the certificate for a claim, then there is a proof of the claim.
- These certificates can be more compact than proofs.
- Generating and checking certificates is easier than generating proofs.
- Proof generation (including LCF) and verification are subsumed.
- Verifying the checkers is (a lot) easier than verifying the inference procedures.

## PVS Festschrift

- PVS won the CAV award this year
- PVS was formally introduced at FME 93
- In 2014 it turns 21 (old enough to drink)
- We are planning a Festschrift for 2014:
  - Still in the conceptual stages
  - Have multiple AFM meetings:
    - in Europe at FM (formerly FME)
    - in the US at CAV
    - possibly in Asia
- Papers would be a mix of historical origins, definitive papers, and applications
- **Hope to see you there!**